# INDEFINITE INTEGRAL

Part I

Indefinite Integral is the opposite of derivative

If $\frac{dy}{dx} = f(x)$ ➡ $y = \int f(x)\, dx$

with respect to 'x'

Integral Symbol

Integrand: Function we want to integrate

## METHODS OF INTEGRATION

### 1 SUBSTITUTION

$I = \int f(g(x))\, g'(x)\, dx$ ➡ Let $g(x) = u$ ➡ then $\frac{dg(x)}{dx} = g'(x) = \frac{du}{dx}$

$I = \int f\,(g(x))\; g'(x)\, dx$

$I = \int f\,(u)\, du$

Then we can integrate f(u), and finish by putting g(x) back as u.

### 2 BY PARTS (PRODUCT RULE)

$I = \int f(x).g(x)dx$ ➡ $I = f(x)\int g(x)dx - \int f\,'(x).\left(\int g(x)dx\right)dx$

A helpful rule of thumb is ILATE. Choose f(x) based on which of these comes first:

I L A T E

- **I** – Inverse Trigonometric Functions
- **L** – Logarithmic Functions
- **A** – Algebraic Functions
- **T** – Trigonometric Functions
- **E** – Exponential Functions

### 3 MISCELLANEOUS

Euler's substitutions for integration $I = \int R(x, \sqrt{ax^2 + bx + c}\,)\, dx$

**Substitutions**

1. $\sqrt{ax^2 + bx + c} = t \pm \sqrt{a}x \;\; ; \; a > 0$
2. $\sqrt{ax^2 + bx + c} = \sqrt{a(x - x_1)(x - x_2)} = t\,(x - x_1) = t(x - x_2),$
3. $\sqrt{ax^2 + bx + c} = t\,x \pm \sqrt{c} \;\; ; c > 0.$

# 4 PARTIAL (FRACTION)

Part II

Expressing complicated algebraic fractions into 'Partial Fractions'.

Partial Fractions with 'Repeated Linear Factors' in the denominator.

| Denominator containing | Expression | Form of Partial Fractions |
| --- | --- | --- |
| Linear factor | $\frac{f(x)}{(x+a)(x+b)}$ | $\frac{A}{x+a}+\frac{B}{x+b}$ |
| Repeated linear factors | $\frac{f(x)}{(x+a)^3}$ | $\frac{A}{x+a}+\frac{B}{(x+a)^2}+\frac{C}{(x+a)^3}$ |
| Quadratic term (which cannot be factored) | $\frac{f(x)}{(ax^2+bx+c)(gx+h)}$ | $\frac{Ax+B}{ax^2+bx+c}+\frac{C}{gx+h}$ |

# BASIC INTEGRALS & TRICKS

## ALGABRAIC

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$n \neq -1,\ n \in R$$

$$\int \frac{dx}{ax+b} = \frac{\ln(ax+b)}{a} + C$$

$$\int a^{px+q}\,dx = \frac{a^{px+q}}{p.\ln a}\ ;\ a > 0$$

## TRIGONOMETRIC

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

$$\int \operatorname{cosec}^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + C$$

$$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$$

## MISCELLANEOUS

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}$$

$$\int \frac{dx}{(x-a)\sqrt{(x-a)(b-x)}}$$

Put : $x = a\cos^2\theta + b\sin^2\theta$

$$\int \frac{dx}{\sqrt{(x-a)(x-b)}}$$

Put : $x = a\sec^2\theta - b\tan^2\theta$

$$\int \frac{dx}{(ax+b)\sqrt{px+q}}$$

Put $px+q = t^2$