

# INDEFINITE INTEGRAL

Part I

Indefinite Integral is the opposite of derivative

If  $\frac{dy}{dx} = f(x)$   $\Rightarrow$   $y = \int f(x) dx$  with respect to 'x'

**Integral Symbol** **Integrand: Function we want to integrate**

## METHODS OF INTEGRATION

### 1 SUBSTITUTION

$I = \int f(g(x)) g'(x) dx \Rightarrow$  Let  $g(x) = u \Rightarrow$  then  $\frac{dg(x)}{dx} = g'(x) = \frac{du}{dx}$

$I = \int f(g(x)) g'(x) dx$

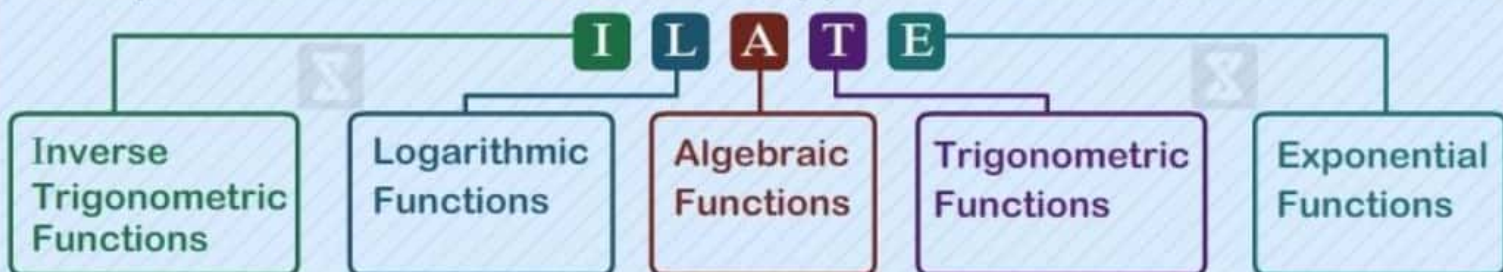
$I = \int f(u) du$

Then we can integrate  $f(u)$ , and finish by putting  $g(x)$  back as  $u$ .

### 2 BY PARTS (PRODUCT RULE)

$I = \int f(x).g(x)dx \Rightarrow I = f(x) \int g(x)dx - \int f'(x). \left( \int g(x)dx \right) dx$

A helpful rule of thumb is **ILATE**. Choose  $f(x)$  based on which of these comes first:



### 3 MISCELLANEOUS

Euler's substitutions for integration

$I = \int R(x, \sqrt{ax^2+bx+c}) dx$

#### Substitutions

1.  $\sqrt{ax^2+bx+c} = t \pm \sqrt{ax}$  ;  $a > 0$

3.  $\sqrt{ax^2+bx+c} = tx \pm \sqrt{c}$  ;  $c > 0$ .

2.  $\sqrt{ax^2+bx+c} = \sqrt{a(x-x_1)(x-x_2)} = t(x-x_1) = t(x-x_2)$ ,



# 4 PARTIAL (FRACTION)

Expressing complicated algebraic fractions into 'Partial Fractions'.  
 Partial Fractions with 'Repeated Linear Factors' in the denominator.

Denominator containing	Expression	Form of Partial Fractions
Linear factor	$\frac{f(x)}{(x+a)(x+b)}$	$\frac{A}{x+a} + \frac{B}{x+b}$
Repeated linear factors	$\frac{f(x)}{(x+a)^3}$	$\frac{A}{x+a} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3}$
Quadratic term (which cannot be factored)	$\frac{f(x)}{(ax^2+bx+c)(gx+h)}$	$\frac{Ax+B}{ax^2+bx+c} + \frac{C}{gx+h}$

## BASIC INTEGRALS & TRICKS

### ALGABRAIC

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$n \neq -1, n \in \mathbb{R}$$

$$\int \frac{dx}{ax+b} = \frac{\ln(ax+b)}{a} + C$$

$$\int a^{px+q} dx = \frac{a^{px+q}}{p \cdot \ln a}; a > 0$$

### TRIGONOMETRIC

$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + C$$

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$$

$$\int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + C$$

$$\int \operatorname{cosec}^2(ax+b) dx = -\frac{1}{a} \cot(ax+b) + C$$

$$\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$$

### MISCELLANEOUS

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a}$$

$$\int \frac{dx}{(x-a)\sqrt{(x-a)(b-x)}}$$

$$\text{Put : } x = a \cos^2 \theta + b \sin^2 \theta$$

$$\int \frac{dx}{\sqrt{(x-a)(x-b)}}$$

$$\text{Put : } x = a \sec^2 \theta - b \tan^2 \theta$$

$$\int \frac{dx}{(ax+b)\sqrt{px+q}}$$

$$\text{Put } px+q = t^2$$